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NATURAL CONVECTIVE HEAT-TRANSFER FROM A VERTICAL SURFACE OF UNIFORM HEAT FLUX TO A NON-NEWTONIAN SUTTERBY FLUID

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Abstract—This paper deals with the laminar natural convection of a non-Newtonian fluid along a vertical surface with uniform heat flux. The boundary layer equations for a Sutterby fluid are solved numerically, and the typical results for the local Nusselt number Nu_x are represented graphically. From the results an approximate expression of Nu_x is proposed as

$$Nu_x = 0{\cdot}62(Gr^*_{0x}Pr_0)^{0{\cdot}2(1+m^*)}.$$

where

$$m^* = 0{\cdot}06Pr_0^{-0{\cdot}28}A^{3{\cdot}7\,Pr_0^{-0{\cdot}34}}\,Z_0^{*\,0{\cdot}35A^{0{\cdot}66}},$$

Gr^*_{0x} and Pr_0 are Grashof and Prandtl numbers based on zero viscosity respectively, and A and Z^*_0 are non-Newtonian parameters.

Local heat-transfer coefficients are obtained by experiments with aqueous solutions of polyethyleneoxide (PEO) and carboxymethylcellulose (CMC). The experimental results are in excellent agreement with the theoretical predictions.

NOMENCLATURE

- A, B, constants in the Sutterby model (1) [–], [s];
- Gr^*_{0x}, Grashof number for a Sutterby fluid defined by (17);
- Gr^*_x, Grashof number for a Newtonian fluid;
- g, gravitational acceleration [m/s^2];
- m^*, exponent in (20);
- Nu_x, local Nusselt number defined by (16);
- Pr_0, Prandtl number for a Sutterby fluid defined by (13);
- q_w, uniform heat flux at the heated surface [W/m^2];
- R, viscosity ratio defined by (12);
- T, temperature [°C];
- U, V, dimensionless velocity components in X- and Y- directions defined by (9) and (10) respectively;
- u, v, velocity components in x- and y-directions respectively [m/s];
- X, Y, dimensionless coordinates for x and y defined by (7) and (8) respectively;
- x, vertical distance from the leading edge of the heated surface [m];
- y, distance normally away from the heated surface [m];
- Z^*_0, dimensionless parameter defined by (14).

Greek symbols

- α_x, local heat-transfer coefficient based on $(T_w - T_\infty)$ [W/m^2 deg];
- β, average coefficient of thermal expansion [1/deg];
- $\dot{\gamma}$, shear rate [1/s];
- Θ, dimensionless temperature defined by (11);
- κ, thermal diffusivity [m^2/s];
- λ, thermal conductivity [W/m deg];
- μ, dynamic viscosity of a Newtonian fluid [Ns/m^2];
- μ_{app}, apparent dynamic viscosity expressed by (1) [Ns/m^2];
- μ_0, constant in the Sutterby model (1) (zero viscosity) [Ns/m^2];
- ν_0, kinematic zero viscosity $= \mu_0/\rho$ [m^2/s];
- ρ, density [kg/m^3];

Subscripts

- w, values at the heated surface;
- ∞, values in the ambient fluid.

1. INTRODUCTION

IN A PREVIOUS paper [1] the authors presented a numerical non-similarity solution of the boundary layer equations for the laminar natural convection of a Sutterby fluid along a vertical isothermal surface. The theoretical predictions exhibited excellent agreement with local Nusselt numbers obtained by experiments with aqueous solutions of polyethyleneoxide (PEO) and carboxymethylcellulose (CMC). In the present study the case of uniform heat flux is treated by using the same method of analysis, the same experimental apparatus and the same fluids.

As for the latter case only a few studies are available in the literature. Tien [2] obtained an approximate solution by using the integral method for a power law fluid. In his analysis the inertia term in the momentum equation is ignored and, therefore, the results are somewhat uncertain. Dale and Emery [3] made a numerical analysis for a power law fluid, and measured the local heat transfer, temperature and velocity profiles with aqueous solutions of CMC and carboxypolymethylene. Although they assessed the effects of varying flow indices and of varying departure point below which shear rate the fluid has a Newtonian behavior upon temperature and velocity profiles, their calculations were restricted within a range of shear rate where these effects have little influence on the numerical results.

The Sutterby model used in the present study describes non-Newtonian behavior accurately, and facilitates the straight forward calculation.

2. ANALYSIS

The physical system and coordinates are shown in Fig. 1, where x is the vertical distance from the leading edge of the heated surface, y the distance normally away from the heated surface, u and v velocity components in x- and y-directions respectively, T temperature, and q_w surface heat flux. Apparent viscosity

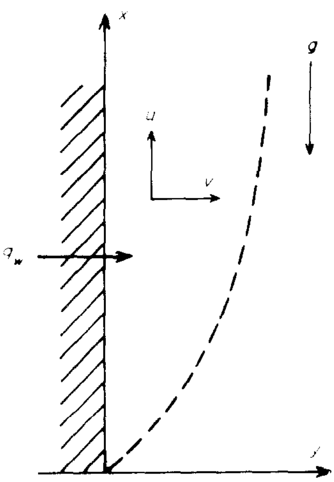

FIG. 1. Physical system and coordinates.

μ_{app} of non-Newtonian fluid is assumed to be expressed by the Sutterby model that

$$\mu_{app} = \mu_0 \left(\frac{\text{arc sinh } B\dot{\gamma}}{B\dot{\gamma}} \right)^A \tag{1}$$

where A, B and μ_0 are model constants and $\dot{\gamma}$ is shear rate.

The dimensionless boundary layer equations for the pertinent problem are written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$\frac{1}{Pr_0}\left(U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} \right) = \Theta + \frac{\partial}{\partial Y}\left(R\frac{\partial U}{\partial Y} \right) \tag{3}$$

$$U\frac{\partial \Theta}{\partial X} + V\frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial Y^2} \tag{4}$$

with boundary conditions of

$$U = V = 0, \quad \frac{\partial \Theta}{\partial Y} = -1 \quad \text{at } Y = 0, \tag{5}$$

$$U = 0, \quad \Theta = 0 \quad \text{at } Y = \infty, \tag{6}$$

where

$$X = \frac{x}{(B\nu_0)^{\frac{1}{2}}} \tag{7}$$

$$Y = \frac{y}{(B\nu_0)^{\frac{1}{2}}} Z_0^{*\frac{1}{5}} Pr_0^{\frac{1}{5}} \tag{8}$$

$$U = u\frac{(B\nu_0)^{\frac{1}{2}}}{\nu_0} Z_0^{*-\frac{2}{5}} Pr_0^{\frac{3}{5}} \tag{9}$$

$$V = v\frac{(B\nu_0)^{\frac{1}{2}}}{\nu_0} Z_0^{*-\frac{1}{5}} Pr_0^{\frac{4}{5}} \tag{10}$$

$$\Theta = \frac{T - T_\infty}{q_w(B\nu_0)^{\frac{1}{2}}/\lambda} Z_0^{*\frac{1}{5}} Pr_0^{\frac{1}{5}} \tag{11}$$

$$R = \frac{\mu_{app}}{\mu_0} \tag{12}$$

$$Pr_0 = \frac{\nu_0}{\kappa} \tag{13}$$

$$Z_0^* = \frac{(B\nu_0)^2 g\beta q_w}{\lambda \nu_0^2} \tag{14}$$

and $B\dot{\gamma}$ in R is written as

$$B\dot{\gamma} = Z_0^{*\frac{3}{5}} Pr_0^{-\frac{2}{5}} \frac{\partial U}{\partial Y}. \tag{15}$$

Average coefficient of thermal expansion β, density ρ, apparent viscosity μ_{app} and thermal diffusivity κ may be evaluated at each appropriate reference temperature.

Local Nusselt number Nu_x is given by

$$Nu_x = \frac{\alpha_x x}{\lambda} = \frac{1}{\Theta_w} X(Z_0^* Pr_0)^{\frac{1}{5}}$$

$$= \frac{1}{\Theta_w} X^{\frac{1}{5}}(Gr_{0x}^* Pr_0)^{\frac{1}{5}} \quad (16)$$

where generalized local Grashof number Gr_{0x}^* is defined by

$$Gr_{0x}^* = \frac{x^4 g\beta q_w}{\lambda \nu_0^2}. \quad (17)$$

It is seen from (2)–(16) that Nu_x is a function of four parameters $Gr_{0x}^* Pr_0$, Pr_0, A and Z_0^* similarly to the case of isothermal surface.

Equations (2)–(4) are solved numerically by the same method as referred in a previous paper [4]. In the present case, however, the heat flux at the surface and the local Nusselt number are described as follows;

$$\left(\frac{\partial \Theta}{\partial Y}\right)_w = \frac{1}{2\Delta Y}(-3\Theta_{m+1,1} + 4\Theta_{m+1,2} - \Theta_{m+1,3})$$

$$= -1 \quad (18)$$

and

$$Nu_x = \frac{3}{4\Theta_{m+1,2} - \Theta_{m+1,3} + 2\Delta Y} \times X^{\frac{1}{5}}(Gr_{0x}^* Pr_0)^{\frac{1}{5}} \quad (19)$$

where $(m + 1)$ and n in $\Theta_{m+1,n}$ represent the X and Y points of the nodal points.

First, the computation was made on the Newtonian fluid of $A = 0$ and $Pr_0 = 100$ in order to test the accuracy of the method of numerical analysis. The obtained local Nusselt number showed good agreement with the corresponding similarity solution of Sparrow and Gregg [5] within the accuracy of 2·5 per cent in the range of $Gr_x^* Pr \geqslant 10^9$.

Generally Nu_x for the non-Newtonian fluid is higher than that for the Newtonian fluid of $\mu = \mu_0$. The effect of A, Z_0^* and Pr_0 on the relation of Nu_x vs $Gr_{0x}^* Pr_0$ are shown in Figs. 2(a), (b) and (c) respectively. There is seen the same tendency as the case of isothermal surface.

By the modification of the relation of Nu_x vs $Gr_x^* Pr$ for a Newtonian fluid of large Prandtl number, the numerical results are correlated approximately as

$$Nu_x = 0{\cdot}62(\mathrm{Gr}_{0x}^* Pr_0)^{0\cdot 2(1+m^*)} \quad (20)$$

where

$$m^* = 0{\cdot}06 Pr_0^{-0\cdot 28} A^{3\cdot 7 Pr_0^{-0\cdot 34}} Z_0^{*0\cdot 35 A^{0\cdot 66}}. \quad (21)$$

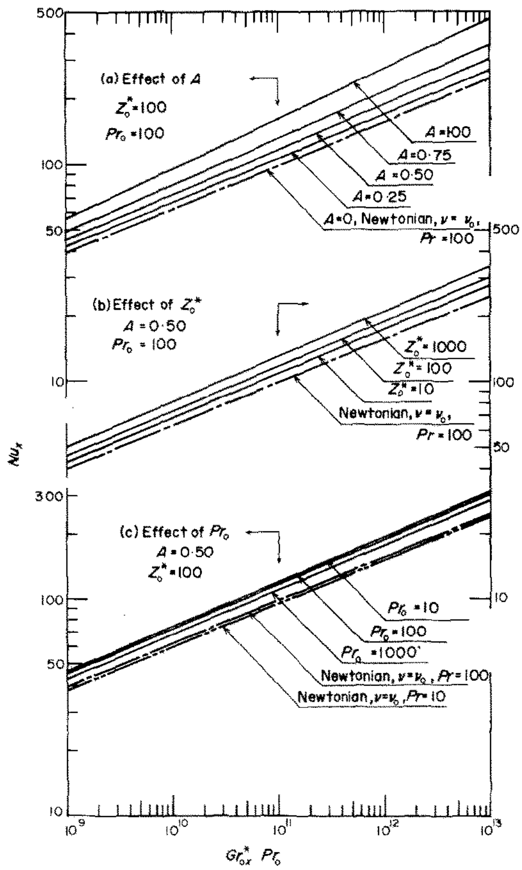

FIG. 2. Variation of local Nusselt number with the product of local Grashof number and Prandtl number.
(a) effect of A.
(b) effect of Z_0^*.
(c) effect of Pr_0.

Expression (20) predicts the local Nusselt number within the accuracy of ± 5 per cent in the ranges of $A = 0$–1, $Z_0^* = 0$–10^4, $Pr_0 = 10^2$–3×10^3 and $Gr_{0x}^* Pr_0 = 10^9$–10^{13}. The comparison between (21) and theoretical results evaluated at $Gr_{0x}^* Pr_0 = 8 \times 10^{10}$ for $Pr_0 = 1000$ is shown in Fig. 3.

3. EXPERIMENTS

The experimental apparatus was the same as that for the case of isothermal surface, that is, the heated surface was a vertical cylinder of 1000 mm height and 82·0 mm dia. The cylinder which subdivided into 20 parts was heated from inside by each corresponding 20 heaters. The fluids used were aqueous solutions of 0·5 and 0·2% PEO and 2·0% CMC.

A series of measurements were carried out for the uniform heat flux conditions and the isothermal conditions [1] alternatively by increasing the electric inputs to 20 heaters gradually. The electric inputs to

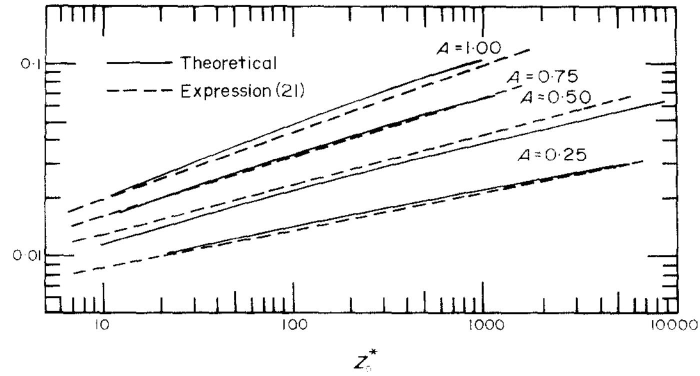

FIG. 3. Comparison of exponent m^* of (21) with calculated values.

the lowest and upper two heaters were over-supplied by the amount of conductive heat loss through the supporting assembly of the heated cylinder and the electric lead wires. The measured values corresponding to these sections, therefore, were not taken up in the correlated data. The deviation of each heat flux of other sections from the average one was confirmed to be within $\pm 1{\cdot}5$ per cent.

The model constants of fluids were measured by authors, and other physical properties were assumed to be the same as those of pure water. Experimental conditions and model constants are shown in Table 1.

Table 1. Conditions of experiments

Run	Symbol	Fluid	q_w(W/m^2)	T_∞ (°C)	A	Z_0^*	Pr_0
1	○	0·2% PEO	1150	19·8	0·3	13	140
2	□	0·2% PEO	2340	16·2	0·3	24	130
3	△	0·2% PEO	6230	16·8	0·3	74	90
4	●	0·5% PEO	2340	23·2	0·4	2400	2600
5	■	0·5% PEO	4650	23·1	0·4	5100	1700
6	×	2·0% CMC	2310	23·3	0·5	0·0027	670
7	+	2·0% CMC	4630	22·0	0·5	0·0055	490

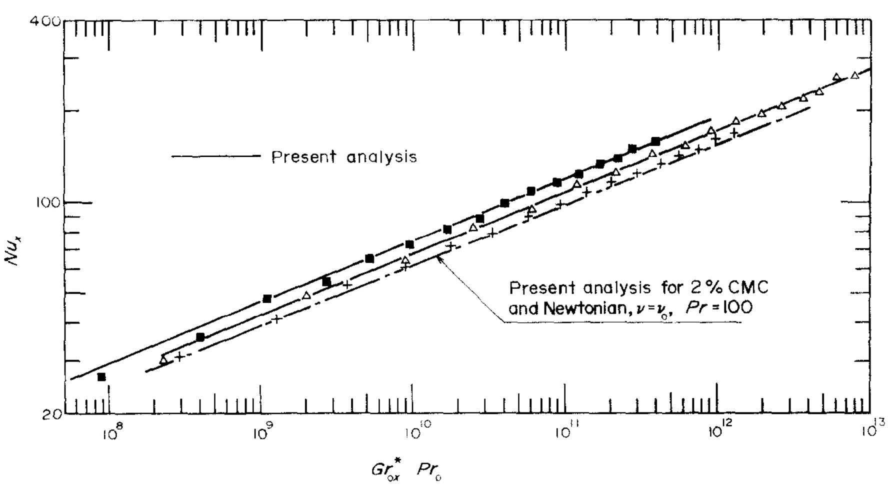

FIG. 4. Comparison between experimental results and present analysis. Symbols correspond to those in Table 1.

The range of shear rate induced was estimated to be the same order as that in the case of isothermal condition. Coefficient of thermal expansion β was taken as the average one evaluated in the temperature range from T_∞ to $(T_w + T_\infty)/2$, and the other physical properties κ, λ, μ_0 and ρ were evaluated at reference temperature $T_w - 0{\cdot}25(T_w - T_\infty)$, where T_w was local value.

Three examples of local Nusselt number are shown in Fig. 4, where each symbol corresponds to that in Table 1. Theoretical solutions corresponding to each experimental condition are also shown in the same figure. The agreement between theory and experiments is excellent.

In Fig. 5 all data are plotted in the relation of Nu_x vs $(Gr_{0x}^{*}Pr_0)^{(1+m^*)}$. Approximate expression (20) is also in good agreement with the measured values within the accuracy of ± 10 per cent.

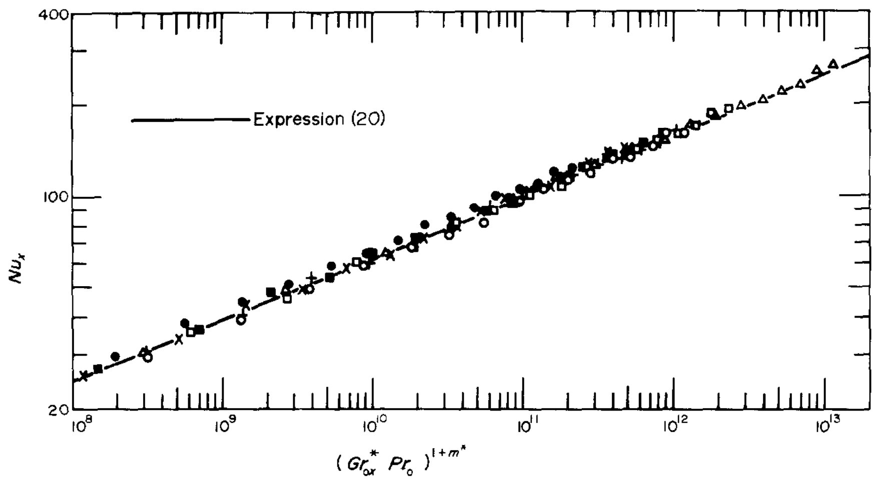

FIG. 5. Comparison between experimental results and approximate expression (20). Symbols correspond to those in Table 1.

4. CONCLUSIONS

(1) Natural convection of a non-Newtonian Sutterby fluid along a surface of uniform heat flux is analyzed numerically, and the local heat-transfer coefficient is obtained with sufficient accuracy. The effect of non-Newtonian parameters A, Z_0^* and Pr_0 upon local Nusselt number is quite similar to that for the case of uniform surface temperature.

(2) Experimental data on Nu_x are in excellent agreement with the numerical analysis and they are also in good agreement with approximate expression (20).

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CONVECTION NATURELLE PAR UN FLUIDE NON-NEWTONIEN DE SUTTERBY SUR UNE SURFACE VERTICALE CHAUFFÉE A FLUX CONSTANT

Résumé—On traite de la convection naturelle laminaire par un fluide non-newtonien le long d'une surface verticale avec flux de chaleur constant. On résout numériquement les équations de couche limite pour un fluide de Sutterby et on représente graphiquement les résultats typiques relatifs au nombre de Nusselt local Nu_x. On propose pour expression approchée des résultats:

$$Nu_x = 0.62(Gr^*_{0x}\, Pr_0)^{0.2(1+m^*)}$$

où

$$m^* = 0.06 Pr_0^{-0.28} A^{3.7 Pr_0^{-0.34}} Z_0^{*0.35} A^{0.66}$$

Gr_{0x} et Pr_0 sont les nombres de Grashof et de Prandtl basés respectivement sur la viscosité d'indice zéro et A et Z_0^* sont des paramètres non-newtoniens.

On obtient expérimentalement les coefficients de convection locaux pour des solutions aqueuses d'oxyde de polyéthylène (PEO) et de carbométhylcellulose (CMC). Les résultats expérimentaux sont en excellent accord avec les estimations théoriques.

WÄRMEÜBERGANG BEI NATÜRLICHER KONVEKTION VON EINER VERTIKALEN OBERFLÄCHE MIT GLEICHMÄSSIGEM WÄRMEFLUSS AN EIN NICHT—NEWTONSCHES "SUTTERBY—FLUID"

Zusammenfassung—Dieser Artikel behandelt die laminare natürliche Konvektion eines nicht—Newtonschen Fluids, längs einer vertikalen Oberfläche mit gleichmässigem Wärmefluss. Die Grenzschichtgleichungen für ein "Sutterby-Fluid" werden numerisch gelöst, und die typischen Ergebnisse für die lokale Nusselt-Zahl Nu_x werden graphisch dargestellt. Aus den Ergebnissen wird eine Näherungsbeziehung vorgeschlagen.

$$Nu_x = 0.62\,(Gr^*_{0x} Pr_0)^{0.2(1+m^*)}$$

wobei

$$m^* = 0.06\, Pr_0^{-0.28} A^{3.7\, Pr_0^{-0.34}} Z_0^{*0.35} A^{0.66}$$

Gr^*_{0x} und Pr_0 sind Grashof und Prandtl-Zahlen, jeweils auf Viskosität Null bezogen, und A und Z^*_0 sind nicht-Newtonsche Parameter. Lokale Wärmeübergangskoeffizienten werden aus Experimenten mit wassrigen Lösungen Polyäthylenoxid (PEO) und "Carboxymethylcellulose" (CMC) erhalten. Die experimentellen Ergebnisse zeigen ausgezeichnete Übereinstimmung mit theoretischen Voraussagen.

СВОБОДНОКОНВЕКТИВНЫЙ ПЕРЕНОС ТЕПЛА ОТ ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ В НЕНЬЮТОНОВСКОЙ ЖИДКОСТИ САТТЕРБИ ПРИ ПОСТОЯННОМ ТЕПЛОВОМ ПОТОКЕ

Аннотация—В статье рассматривается ламинарная естественная конвекция при обтекании вертикальной поверхности неньютоновской жидкостью с постоянным тепловым потоком. Численно решаются уравнения пограничного слоя для жидкости Саттерби, и графически представлены типичные результаты для локального числа Нуссельта Nu_x

На основании полученных данных предложено следующее приближенное выражение для Nu_x:

$$Nu_x = 0{\cdot}62\,(Gr^*_{0x}\, Pr_0)^{0{\cdot}2\,(1+m^*)}$$

где

$$m^* = 0{\cdot}06\, Pr_0^{-0{\cdot}28} A^{3{\cdot}7\, Pr_0^{-0{\cdot}34}} Z_0^{*0{\cdot}35} A^{0{\cdot}66}$$

$Gr_{0x}{}^*$ и Pr_0—числа Грасгофа и Прандтля, расчитанные при нулевой вязкости, а A и $Z_0{}^*$—неньютоновские параметры.

Локальные коэффициенты теплообмена получены экспериментально для водных растворов полиэтиленоксида [ПЭО] и карбоксиметилцелюллозы [КМЦ]. Экспериментальные результаты хорошо согласуются с теоретическими расчетами.